

## DYNAMIC-RANGE FIGURE OF MERIT IN HIGH-GAIN AMPLIFIERS

Pertti K. Ikalainen

Central Research Laboratories, Texas Instruments Incorporated  
POB 655936 MS 134, Dallas, TX 75265

### ABSTRACT

An optimum scaling that maximizes the (output and input) two-tone third-order intercept point of an amplifier with respect to dc power consumption is derived. The expression for spurious-free dynamic range of a high-gain amplifier is factored to separate effects of device parameters on dynamic range and to derive a new figure of merit.

### INTRODUCTION

Many microwave receiver applications require low-noise amplifiers with the ability to withstand strong input signals without generating excessive distortion, i.e., the amplifiers must be linear. Linearity in this sense usually is measured by the two-tone, third-order input or output intercept point (*IIP* and *OIP*) [1]. However, low dc power consumption may be equally important. High dynamic range, low noise figure, and low power consumption are all desirable, but unfortunately conflicting, goals in receiver design. Therefore, when considering a particular device for receiver applications, one must take into account at least four performance parameters: gain, noise figure, *OIP*, and dc power consumption. Useful figures of merit that combine several performance factors into a single number in a meaningful way exist for different modes of operation of active devices, such as the noise measure for low-noise operation or power-added efficiency for high-power operation. No such measure exists, however, for dynamic range. This paper develops a new, physically meaningful figure of merit for dynamic range. It is used to compare recent high dynamic range devices on an unequivocal basis. Also, an optimum scaling that maximizes *OIP* and *IIP* of an amplifier with respect to dc power is derived.

### OPTIMUM SCALING FOR INTERCEPT POINT

Because the successive stages of a multistage amplifier work with higher and higher levels of signal power, it is intuitively clear that devices toward the output should have higher intercept points (i.e., they either should be more linear devices or have larger gate width, emitter length, etc.). However, to save power device size should be kept to a minimum. Therefore, there exists an optimum scaling between the stages that will

maximize the overall *OIP/P<sub>dc</sub>* ratio which is sometimes used as a figure of merit [2].

Consider the cascading formula for the intercept point of a multistage amplifier [1]

$$\frac{1}{IIP_t} = \frac{G_1 G_2 \cdots G_N}{OIP_t} = \frac{1}{IIP_1} + \frac{G_1}{IIP_2} + \frac{G_1 G_2}{IIP_3} + \cdots + \frac{G_1 G_2 \cdots G_{N-1}}{IIP_N}. \quad (1)$$

Incidentally, a convenient verbal interpretation of this formula is that *IIP*s are added as "resistances in parallel" with all the *IIP*s referenced to the input, i.e., the *IIP* of the second and succeeding stages are reduced by the gain of the preceding stages. Assuming that the intercept points are proportional to the size of a device (for a given type of device at a given relative bias that can be expressed in, for example, mA/mm), and making use of the *OIP/P<sub>dc</sub>* figure of merit [2] we can express the *IIP* and *OIP* of each stage as

$$IIP_i = \frac{OIP_i}{G_i} = \alpha_i P_{dc,i} = \frac{\beta_i P_{dc,i}}{G_i}. \quad (2)$$

$\alpha_i$  and  $\beta_i$  will be called the input and output linearity efficiency, respectively. In the case of a two-stage amplifier, we can write from (1) and (2), see Fig. 1,

$$\frac{P_{dc1} + P_{dc2}}{IIP_t} = \frac{P_{dc1} + P_{dc2}}{\alpha_1 P_{dc1}} + \frac{G_1 (P_{dc1} + P_{dc2})}{\alpha_2 P_{dc2}}. \quad (3)$$

The left side of (3) can be minimized with respect to  $P_{dc2} / P_{dc1}$  to give the optimum ratio between device sizes

$$\frac{P_{dc2}}{P_{dc1}} = \sqrt{\frac{G_1 \alpha_1}{\alpha_2}} = \sqrt{\frac{\beta_1}{\alpha_2}}. \quad (4)$$

For the same kind of device in both stages, the scaling formula says they should be scaled by the square root of gain. It can be shown that the  $\sqrt{G}$  rule is valid for any number of stages if the devices are of the same kind (only size is scaled).

IF1

## DYNAMIC-RANGE FIGURE OF MERIT

In any specific case involving a certain kind of device, it is easy to determine the dynamic range of an N-stage amplifier. However, if we try to compare different kinds of devices, things get more complicated. For example, a high-gain device would appear to give less dynamic range, all other parameters (including the number of stages) being equal. However, because fewer stages are required for a certain gain, then, from a system point of view, the high-gain device may be better. A meaningful figure of merit must separate system design parameters from device parameters. This can be done in the case of high-gain amplifiers, as will be shown in the following.

Consider an amplifier with N stages of otherwise identical amplifying devices but the size of the device being scaled with the  $\sqrt{G}$  rule from stage to stage to obtain the highest possible  $OIP/P_{dc}$  ratio for the amplifier. Noise figure and gain are assumed to be the same for each stage, as they do not depend strongly on device size, while intercept points and dc power are assumed to scale linearly with size. (The noise figure of an FET may depend on size if unit gate width is scaled instead of the number of fingers. Also, experimental noise figures depend on device size at a given frequency because of varying matching circuit losses depending on location of  $\Gamma_{opt}$ . These subtleties are ignored in this analysis aimed at deriving a figure of merit.) Intermodulation power is assumed vary 3 dB for every 1-dB change in input power. Defining the dynamic range to extend from the noise floor to the level where intermodulation products first appear from noise, we get, under the assumptions stated above,

$$d = \left( \frac{IIP_t}{P_n} \right)^{\frac{1}{2}}, \quad (5)$$

where  $IIP_t$  is the input intercept point and  $P_n$  is the noise floor of the amplifier, respectively.  $P_n$  is given by

$$P_n = kB((F_t - 1)T_0 + T_s), \quad (6)$$

where  $k$  is the Boltzmann constant,  $B$  system bandwidth,  $F_t$  noise figure of the amplifier,  $T_0$  standard reference temperature (290 K), and  $T_s$  is the noise temperature of the input termination connected to the amplifier. Using cascading formulas for noise figure and input intercept point and expressing intercept points in terms of dc power with the aid of linearity efficiency (see Eq. (2)) we get for the dynamic range of an N-stage amplifier

$$d^{\frac{1}{2}} = \frac{P_{det}}{kBT_0G^N} \frac{\beta}{G} \left( \frac{\sqrt{G} - 1}{\sqrt{G} - 1} \right)^2 \times \frac{1 - G^{-1}}{(F - 1)(1 - G^{-N}) + \frac{T_s}{T_0}(1 - G^{-1})}, \quad (7)$$

where  $G$  is gain,  $F$  noise figure, and  $\beta$  output linearity efficiency ( $OIP/P_{dc}$ ) of the devices in the amplifier, respectively, and  $P_{det}$  is the total power consumption. As stated before, devices were assumed to be scaled with the  $\sqrt{G}$  rule to obtain the optimum  $OIP/P_{dc}$  ratio.

If the total gain  $G_t = G^N$  of the amplifier is high, either by cascading a large number of stages or by high individual stage gains, we can set  $G^{-N} \approx 0$  in (7). The error introduced is less than 1 dB for most practical values of  $F$  and  $T_s$  if  $G_t > 16.5$  dB. We will now consider two specific cases, those of  $T_s$  being equal to  $T_0$  and  $T_s$  being 0 K. In the first case (7) reduces to

$$D \approx \frac{2}{3} \times 10 \log \left( \frac{P_{det}}{kT_0BG_t} \right) + \frac{2}{3} \times 10 \log \left( \frac{\beta(1 - \sqrt{G^{-1}})^2(1 - G^{-1})}{F - G^{-1}} \right), \quad (8)$$

where we now express the dynamic range in dB. The first factor in (8) depends only on system parameters such as bandwidth, total gain, and dc power consumption. The second part contains factors specific to the device: gain, noise figure, and linearity efficiency. This factor can be considered to be a device-specific dynamic-range figure of merit for the case of the noise temperature of the input termination being 290 K. It shows the effect of *device parameters* on the dynamic range of a high-gain amplifier constructed using a certain kind of device, other system parameters (bandwidth, total gain, and dc power) being equal. We, therefore, define the figure of merit  $Q_D$  (290) as

$$Q_D(290) = \frac{2}{3} \times 10 \log \left( \frac{\beta_{\infty}}{F_{\infty}} \right), \quad (9)$$

where

$$\beta_{\infty} = \beta(1 - \sqrt{G^{-1}})^2, \quad (10)$$

and

$$F_{\infty} = \frac{F - G^{-1}}{1 - G^{-1}}. \quad (11)$$

$\beta_\infty$  can be shown to be the overall  $OIP/P_{dc}$  ratio of an infinite cascade of otherwise identical stages, but scaled with the  $\sqrt{G}$  rule, while  $F_\infty$  is the overall noise figure of an infinite cascade [3].

In the second case,  $T_s=0$  K a figure of merit  $\mathcal{Q}_D(0)$  can be derived from (7) as

$$\mathcal{Q}_D(0) = \frac{2}{3} \times 10 \log \left( \frac{\beta_\infty}{M} \right), \quad (12)$$

where  $M$

$$M = \frac{F-1}{1-G^{-1}} \quad (13)$$

is the noise measure of the device [4].

Figure 2(a) shows the variation of  $\mathcal{Q}_D(290)$  as a function of device gain with noise figure as a parameter. In Fig. 2(a)  $\beta=1$ , but the effect of  $\beta \neq 1$  is simple to take into account as  $\beta$  is a multiplier in  $\mathcal{Q}_D$ . Figure 2(b) shows  $\mathcal{Q}_D(0)$  in a similar fashion.

Table 1 shows experimental data for the gain, noise figure, and  $OIP/P_{dc}$  of some recent low-noise and high-intercept-point devices at 10 GHz\*. Table 2 shows the noise figure,  $OIP/P_{dc}$ , and dynamic range of a multistage amplifier constructed using each type of device, respectively, and scaling successive devices with the  $\sqrt{G}$  rule to obtain the highest possible  $OIP/P_{dc}$ . The data in Table 2 have been computed from the data in Table 1 using formulas discussed in the preceding. It can be observed from Table 2 that the difference between  $\mathcal{Q}_D(290)$  and  $\mathcal{Q}_D(0)$  is largest in the case of the low-noise HEMT which reflects the fact that the contribution of the noise from the input termination becomes dominant for very low noise figures. Table 2 shows the pulse-doped FET being the best high-dynamic-range device with its relatively low noise figure and record high linearity efficiency. Other factors to be considered in evaluating devices include the overall noise figure, number of stages required for a given gain, reliability, ease of matching and bias circuit design, and compatibility with other technologies to be used in the system. It should be noted that this dynamic-range figure of merit applies to one kind of device at one bias point and reflects the general potential of the device for high-dynamic-range applications. Better amplifier performance is achievable by biasing front-end devices for a lower noise figure and final stages for a higher  $OIP$ , or by mixing different types of devices. The optimum scaling formula is still valid, even with different bias conditions or mixed devices and can be used as a design rule.

## CONCLUSION

In a two-stage amplifier, the devices should be scaled by the square root of their  $OIP$  and  $IIP$  to  $P_{dc}$  ratios for the highest  $IIP$  and  $OIP$  to  $P_{dc}$  ratio in the amplifier. In the case of identical devices, the scaling is  $\sqrt{G}$ . The expression for the spurious-free dynamic range of a high-gain amplifier consisting of identical devices scaled in the optimum manner can be factored to derive a dynamic-range figure of merit. Based on this new figure, the low-noise, linearized pulse-doped FET is presently the best high-dynamic-range device.

## REFERENCES

- [1] Tsui, *Microwave receivers and related components*, National Technical Information Center, 1982; Peninsula, Los Altos, CA, 1985.
- [2] Kim, M.E. et al., "GaAs heterojunction bipolar transistor device and IC technology for high-performance analog and microwave applications," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-37, pp. 1286-1303, 1989.
- [3] Duh, K.H.G. et al., "W-band InGaAs HEMT low noise amplifiers," *1990 IEEE International Microwave Symposium Digest*, pp. 595-598.
- [4] Haus, H.A. and Adler, R.B., *Circuit theory of linear noisy networks*. John Wiley & Sons, New York, 1959.
- [5] Ikkalinen, P.K., and Witkowski, L.C., "High dynamic range microwave FET," *Electronics Letters*, vol. 27, No.11, pp. 945-946, May 1991.
- [6] TI internal test data.
- [7] Chu, S.L.G. et al., "A highly linear MESFET," *IEEE MTT-S Int. Microwave Symp. Dig.*, 1991, pp. 725-728.

\* Good  $OIP/P_{dc}$  results were reported in [7] but, as the corresponding noise figure was not disclosed, the figure of merit can not be computed.

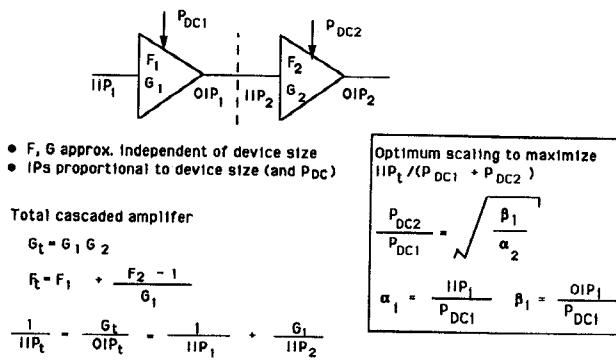


Fig. 1. Cascading formulas for noise figure and *IIP* are shown for the two-stage amplifier case. The inset shows the ratio of stage one to stage two DC power that will result in the highest overall *IIP* and *OIP* to  $P_{dc}$  ratio.

Type of Device	$G$ (dB)	$F$ (dB)	$OIP/P_{dc}$	Ref.
PD MESFET	9.1	3.15	69	[5]
HBT	7.0	4.9	43.9	[2]
HEMT	12.7	0.8	3.4	[6]

Table 1. Experimental gain, noise figure, and *OIP/Pdc* at 10 GHz for some recent low-noise and high-intercept-point devices.

Type of Device	$F_\infty$ (dB)	$\beta_\infty$	$Q_D(290)$ (dB)	$Q_D(0)$ (dB)
PD MESFET	3.5	29.1	7.5	9.2
HBT	5.6	13.4	3.8	4.7
HEMT	0.84	2.0	1.5	6.5

Note: If, for example,  $G_t=20$  dB,  $P_{dc}=10$  mW, and  $B=1$  MHz, then dynamic range is  $D=76$  dB +  $Q_D$ .

Table 2. Noise figure, overall *OIP/Pdc*, and dynamic-range figure of merit for a high-gain amplifier constructed using each type of device, respectively, and scaling successive stages with the  $\sqrt{G}$  rule.

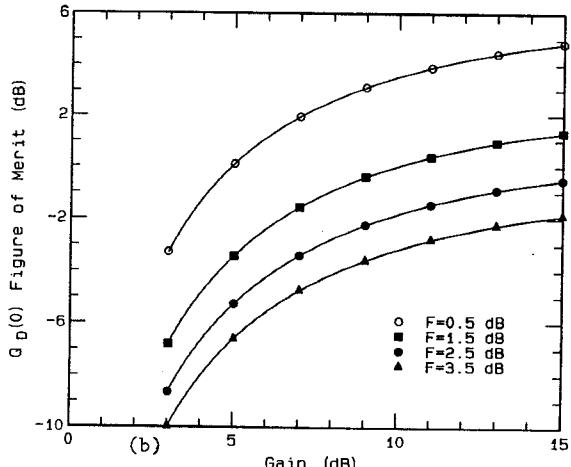
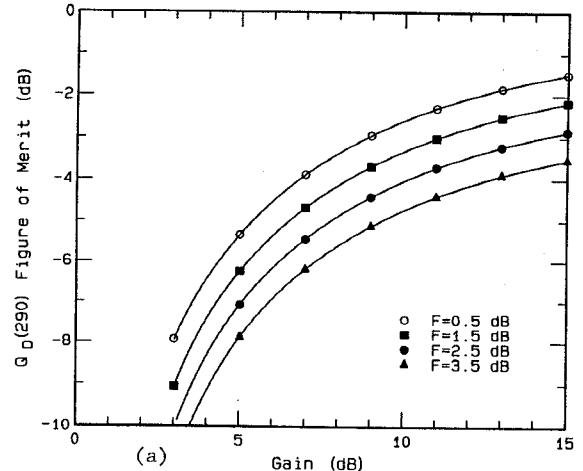


Fig. 2. (a) shows variation of  $Q_D(290)$  as a function of gain with noise figure as a parameter for  $\beta = 1$ . (b) shows  $Q_D(0)$  in the same way.